3.5 Applications of Extrema (Minimum Homework: 1, 3, 5)

We need to remember a few concepts before we start this section.

We covered price-demand equations in section 2.5. We will need to remember this concept. I cut and pasted this write-up from section 2.5

Price-demand function "explained"

- Demand depends on the price of a product.
- The higher the price, the less the demand. (generally)
- The lower the price, the more the demand. (generally)
- A price demand function describes a relationship between the demand (x) for a product and the price $p(x)$ for the same product.

Here is a simple price-demand function: $p(x) = -1.25x + 50$

a) Let us evaluate the function at $x = 20$ and then explain the number generated.

- $p(20) = -1.25(20) + 50$
- $p(20) = 25$
- $x = 20$ represents the number of units of the product sold.
- $p(20) = 25$ represents the price of the product that will yield 20 units sold.
- The Algebra basically tells us at a price of $25, 20 units of the product will be sold.

b) What is total revenue what will be generated when 20 units are sold at $25?

- $revenue = price * quantity$
- $revenue = 25 * 20 = \$500$
- $500 of revenue will be generated when 20 units are sold for $25 each.

c) Use the price demand function to create a revenue function $R(x)$.

- $revenue = price * quantity$
  - We $will\ use\ p(x) = -1.25x + 50\ to\ represent\ price$
  - We will use x to represent quantity.
  - $R(x) = (-1.25x + 50)(x)$
  - $R(x) = -1.25x^2 + 50x$

d) Use the revenue function to calculate the revenue that will be earned when 20 units are sold.

- $R(20) = -1.25(20)^2 + 50(20) = 500$
- $500 of revenue will be earned when 20 units are sold
- Notice this is the same answer as question b. This is because this is another of way of asking the same question.

We also need the profit formula for this section.

Revenue formula

Given the price demand function $p(x)$
The revenue function $R(x)$ to produce x units of a product is given by the formula:

$$R(x) = xp(x)$$

Profit formula

Profit = Revenue – Cost
$P(x) = R(x) - C(x)$
$where\ P, R\ and\ C\ are\ profit, revenue\ and\ cost\ funtions.$

Let me do the ugliest problem from the section to illustrate the steps needed to solve the problems in this section:

Section 3.5 Problem 6: The Double B Corporation analyzed the production costs for one of its products and determined that the daily cost function can be given by $C(x) = \frac{1}{2}x^2 + 20x + 500$

where x is the number of units produced each day. The price demand function is given by:

$p(x) = -\frac{1}{2}x + 150$

a) Create a revenue function.

$$R(x) = xp(x) = x\left(-\frac{1}{2}x + 150\right) = -\frac{1}{2}x^2 + 150x$$

Answer part a: $R(x) = -\frac{1}{2}x^2 + 150x$

b) Create a profit function.

$$P(x) = R(x) - C(x) = -\frac{1}{2}x^2 + 150x - \left(\frac{1}{2}x^2 + 20x + 500\right)$$

$$P(x) = 150x - \frac{1}{2}x^2 - \frac{1}{2}x^2 - 20x - 500$$

Answer part b: $P(x) = -1x^2 + 130x - 500$

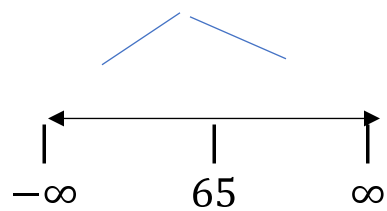
c) How many units must the company produce and sell to maximize profit?

$P'(x) = -2x + 130$

Find critical number:

$-2x + 130 = 0$
$-2x = -130$
$x = 65$

Now confirm $x = 65$ is the x-coordinate of a maximum point.



Interval $(-\infty, 65)$ $check\ x = 0$ $P'(0) = -2(0) + 130 = 130$ $positive, increasing$
Interval $(65, \infty)$ $check\ x = 66$ $P'(66) = -2(66) + 130 = -6$ $negative, decreasing$

$$x = 65\ is\ the\ x - coordinate\ of\ a\ maximum\ point,$$
$$it\ is\ the\ optimal\ quantitiy$$

$Answer\ part\ c\text{:}\ 65\ units$

d) What is the maximum profit?

Substitute 65 for x into the profit function:

Maximum profit: $P(65) = -1(65)^2 + 130(65) - 1500 = 3725$

Answer part d: $maximum\ profit\ \ \$3{,}725$

e) What price per unit must be charged to make maximum profit?

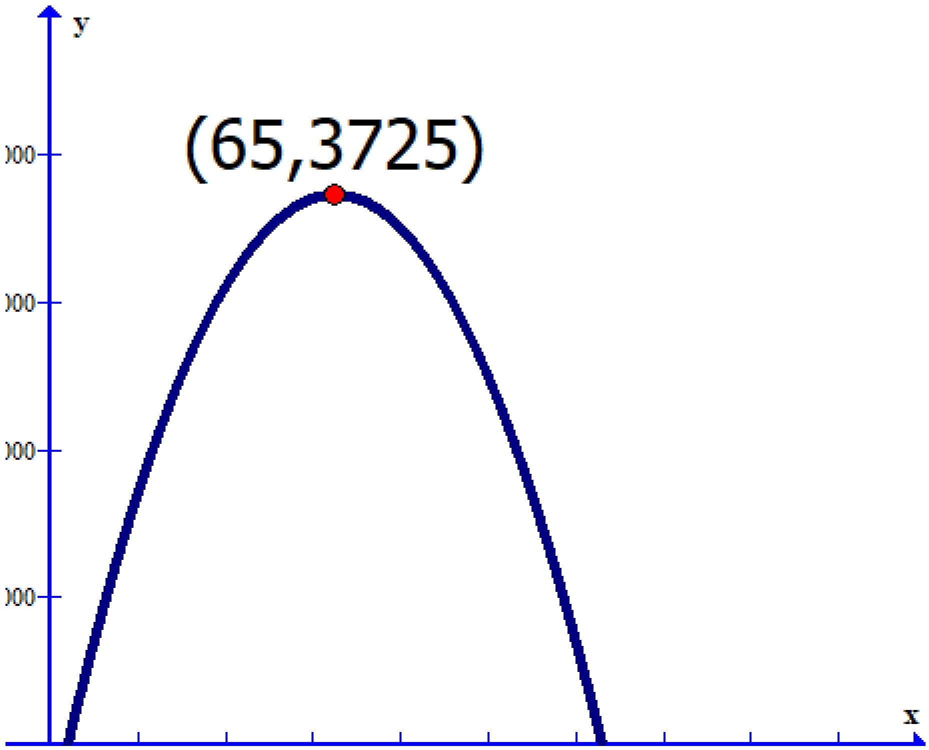
We need to use the price-demand function and let $x = 65$: $p(x) = 150 - \frac{1}{2}x$

$p(65) = 150 - \frac{1}{2}(65) = 117.50$

Answer part e: $optimal\ price\ \$117.50$

Here is a graph of the profit function. You should notice that the optimal quantity of 65, and the maximum profit of \$3,725 occurs at the vertex.

Each point on the graph is of the form: $(number\ of\ units, profit)$



y
(65,3725)
x

1) A company makes a single product. The cost function for the product is given by:
$C(x) = 0.5x^2 + 50x + 200$ where $C(x)$ is the total cost to produce x units of the product.
The demand function is given by $p(x) = -x + 110$, where $p(x)$ is the price to sell x units of the product.

a) Create a revenue function.
b) Create a profit function.
c) How many units must the company produce and sell to maximize profit?
d) What is the maximum profit?
e) What price per unit must be charged to make maximum profit?

2) A company makes a single product. The cost function for the product is given by:

$C(x) = 0.5x^2 + 20x + 200$ where $C(x)$ is the total cost to produce x units of the product.

The demand function is given by $p(x) = -2x + 100$, where $p(x)$ is the price to sell x units of the product.

a) Create a revenue function.

b) Create a profit function.

c) How many units must the company produce and sell to maximize profit?

d) What is the maximum profit?

e) What price per unit must be charged to make maximum profit?

2a) Create a revenue function. $R(x) = -2x^2 + 100x$

2b) Create a profit function. $P(x) = -2.5x^2 + 80x - 200$

2c) How many units must the company produce and sell to maximize profit? $16\ units$

2d) What is the maximum profit? $440

2e) What price per unit must be charged to make maximum profit? $68

3) The marketing research department of Shank, a quarterly magazine for beginning golfers, has determined that the price-demand equation for the magazine is approximated by

$p(x) = -0.1x + 200$

where x represents the number of magazines printed and sold each quarter, in hundreds, and $p(x)$ is the price, in dollars, of the magazine. The cost of printing, distributing, and advertising is given by

$C(x) = 0.2x^2 + 50x + 3000$

a) Create a revenue function.

b) Create a profit function.

c) How many units must the company produce and sell to maximize profit?

d) What is the maximum profit?

e) What price per unit must be charged to make maximum profit?

4) A headphone determines that to sell x units of a new headphone, the price demand equation for the headphones is given by $p(x) = -x + 100$. It also determines that the total cost $C(x)$of producing x units is given by $C(x) = 2x + 50$.

a) Create a revenue function.
b) Create a profit function.
c) How many units must the company produce and sell to maximize profit?
d) What is the maximum profit?
e) What price per unit must be charged to make maximum profit?

4a) Create a revenue function. $R(x) = -1x^2 + 100x$

4b) Create a profit function. $P(x) = -1x^2 + 98x - 50$

4c) How many units must the company produce and sell to maximize profit? $49\ units$

4d) What is the maximum profit? $\$2{,}351$

4e) What price per unit must be charged to make maximum profit? $\$51$

5) The daily production cost $C(x)$for a factory to manufacture x deluxe contour chairs is given to be

$C(x) = \frac{1}{2}x^2 + 14x + 500$. The price demand function is $p(x) = -\frac{3}{2}x + 150$ where $p(x)$ is the price needed to sell $\mathrm{x} - \mathrm{chairs}$.

a) Create a revenue function.

b) Create a profit function.

c) How many units must the company produce and sell to maximize profit?

d) What is the maximum profit?

e) What price per unit must be charged to make maximum profit?

6) The Double B Corporation analyzed the production costs for one of its products and determined that the daily cost function can be given by

$C(x) = \frac{1}{2}x^2 + 20x + 500$

where x is the number of units produced each day. The price demand function is given by:

$p(x) = -\frac{1}{2}x + 150$

a) Create a revenue function.

b) Create a profit function.

c) How many units must the company produce and sell to maximize profit?

d) What is the maximum profit?

e) What price per unit must be charged to make maximum profit?

a) Create a revenue function. $R(x) = -\frac{1}{2}x^2 + 150x$

b) Create a profit function. $P(x) = -1x^2 + 130x - 500$

c) How many units must the company produce and sell to maximize profit? $65\ units$

d) What is the maximum profit? $maximum\ profit\ \ \$3{,}725$

e) What price per unit must be charged to make maximum profit?

$optimal\ price\ \$117.50$